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LETTER TO THE EDITOR

Space reflections in the dichotomic magnetic charge model

A O Barut†, Ya M Shnir‡ and E A Tolkachev‡

† Physics Department, University of Colorado, Boulder, CO 80309, USA
‡ Institute of Physics, 70 F Skorina Avenue, Minsk 220072, Belarus

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Abstract. The arguments in favour of the possibility of the parity-invariant description of the interaction between electric and magnetic charges are re-examined. It is shown that P-invariance is possible if one generalizes the framework of the standard Abelian gauge coupling to a $U(1)\otimes U(1)$ coupling.

It is known that the symmetric electrodynamics with electric and magnetic charges, e and g, violate space (P) parity, if g is a scalar charge like e [1]. A few P-non-invariant physical effects which may be used in the experimental search for magnetic charges were predicted [2]. Therefore any method of restoring the apparent space reflection symmetry violation due to monopoles must be carefully investigated. This is particularly important for two different models of strong interactions that make use of magnetic monopoles [3, 4].

In the present letter we reconsider the arguments of [5, 6] in favour of the possibility of the P-invariant description of the e–g interactions. Introducing further the interaction with the external fields into this theory we show in this work that P-invariance is possible if one generalizes the framework of the $U(1)$ gauge coupling to a $U(1)\otimes U(1)$ coupling.

Let us recall that the key idea of the approach [4] consists in the definition of the magnetic charge operator as a dichotomic variable

$$\mathcal{Q}_g=\begin{pmatrix} g & 0 \\ 0 & -g \end{pmatrix}. \tag{1}$$

The Hamiltonian describing the behaviour of the electric charge (e) in the field of the monopole (g) can then be written as

$$\mathcal{H}=\begin{pmatrix} H(g) & 0 \\ 0 & H(-g) \end{pmatrix} \tag{2}$$

where $H(g)=(\nabla-\mathrm{i}\mu A^{\mathrm{D}})^2/2M$, $\mu=eg$ and

$$A^{\mathrm{D}}=g\tan(\theta/2)\hat{\varphi} \tag{3}$$

is the Dirac potential.

This procedure can be re-expressed as the embedding of the Abelian description of the e–g interaction into an $SU(2)$ description by the definitions of the potential

$$\mathcal{A}=\mathcal{A}^{\mathrm{D}}\sigma_3=\frac{g}{r}\begin{pmatrix} \tan(\theta/2) & 0 \\ 0 & -\tan(\theta/2) \end{pmatrix}\hat{\varphi} \tag{4}$$

where σ_3 is Pauli matrix.

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We note that there are two other motivations for (1) and (2), which are connected with the observability of magnetic charges. One is that the expectation value of $\mathcal{Q}_g$ vanishes. The other is that the magnetic charge must occur in pairs, and by a superselection rule, in the form of parity eigenstates $\{|g\rangle \pm |-g\rangle\}$ [7]. Both of these may point to the difficulties of readily observing magnetic charges. The expression (4) coincides up to a gauge transformation with the Wu–Yang non-Abelian monopole potential [8], which is the solution of the obviously P-invariant Yang–Mills equations of the pure $SU(2)$ model. Therefore a generalized P-inversion operator must exist commuting with the Hamiltonian (2) which is constructed in [5, 6]. Its action on the potential $\mathscr{A}$ and on the wavefunctions is determined by the following equations:

$$\mathscr{P}:\mathscr{A} = S(P\mathscr{A})S^{-1} + \frac{\mathrm{i}}{e}(\nabla S)S^{-1} = \lambda\mathscr{A} \tag{5a}$$

$$\mathscr{P}:\Psi_{Njm\mu}(\boldsymbol{r}) \equiv \mathscr{P}:\begin{pmatrix}\Phi_{Njm\mu}(\boldsymbol{r})\\ \Phi_{Njm-\mu}(\boldsymbol{r})\end{pmatrix} = SP\Psi_{Njm\mu}(\boldsymbol{r}) = \kappa\Psi_{Njm\mu}(\boldsymbol{r}) \tag{5b}$$

where λ, κ are constants, $\lambda^2 = |\kappa^2| = 1$ and $\Psi_{Njm\mu}(\boldsymbol{r})$ are eigenfunctions of the operator (2).

Unlike the usual electrodynamics where it is possible to carry out only (5a), the embedding into $SU(2)$ frame also allows equation (5b). For instance, for $\lambda = -1$ and $\kappa = (-1)^{j-\mu}$, we obtain

$$S = \begin{pmatrix}0 & \mathrm{e}^{-2\mathrm{i}\mu\varphi}\\ \mathrm{e}^{-2\mathrm{i}\mu\varphi} & 0\end{pmatrix}. \tag{6}$$

Thereby $\mathscr{P}:\mathscr{A} = -\mathscr{A}$ and there is indeed the possibility to restore formal P invariance of the interaction between electric and magnetic charges by the above mechanism. But we seem to run into a slight problem if we attempt to include other external electromagnetic fields into this framework together with $\boldsymbol{A}^{\mathrm{D}}$. Indeed, if we are to follow the usual embedding procedure all the electromagnetic potentials have to be multiplied by σ_3. Only in this case is this model effectively equivalent to the $U(1)$ model with the standard rule of adding fields. In this case the non-relativistic Hamiltonian operator of the charge-dyon system would be:

$$\mathscr{H} = -(\boldsymbol{P} + (e\boldsymbol{A}^{\mathrm{D}} + e\boldsymbol{A}^{e})\sigma_3)^2/2M - eA_0^{\mathrm{D}}\sigma_3 - eA_0^3\sigma_3. \tag{7}$$

Instead we can write a P-invariant Hamiltonian operator as

$$\mathscr{H} \doteq -(\boldsymbol{P} + e\boldsymbol{A}^{\mathrm{D}}\sigma_3 + e\boldsymbol{A}^{e}I)^2/2M - eA_0^{\mathrm{D}}\sigma_3 - eA_0^3 I. \tag{8}$$

This, as a matter of fact, implies an extension of electrodynamics because the model with Hamiltonian (8) is invariant under the gauge transformation with $U(1)\otimes U(1)$ group and describes the interaction of the quantum particle carrying pseudoscalar ($e\sigma_3$) and scalar (eI) charges with pseudovector (A_0^{D}, $\boldsymbol{A}^{\mathrm{D}}$) and vector ($A_0^3$, $\boldsymbol{A}^3$) fields, correspondingly. Moreover with this definition it is not necessary to fix the same interaction constant e. Indeed, because Ψ is a two-component entity, the natural gauge transformation is $\Psi \to \exp\{\mathrm{i}(e + e'\sigma_3)\}\Psi$.

Let us illustrate these arguments in the example of the calculation of selection rules for dipole radiation.

It is easy to see that the dipole moment operator corresponding to the Hamiltonian (7) is $e(\boldsymbol{r}\sigma_3)$ and the corresponding operator for the Hamiltonian (8) is $e(\boldsymbol{r}I)$. In the first case the total set includes the charge operator $e\sigma_3$, therefore a matrix element should be calculated between the following type of wavefuntions:

$$\begin{pmatrix} \Psi_{Njm\mu}(\boldsymbol{r}) \\ 0 \end{pmatrix} \qquad \text{or} \qquad \begin{pmatrix} 0 \\ \Psi_{Njm\mu}(\boldsymbol{r}) \end{pmatrix}$$

where $\Psi_{Njm\mu}(\boldsymbol{r}) = R_{Nj}(r)Y_{jmq}(\theta, \varphi)$ are the wavefunctions of the electrically charged particles in the field (3); $R_{Nj}(r)$ is their radial part, j and m are the quantum numbers corresponding to the eigenvalues of the operator J^2 and J_3 ($\boldsymbol{J}$ is the operator for the total angular momentum of the system) and the generalized spherical harmonics $Y_{jmq}(\theta, \varphi)$ are expressed through the standard Jacobi polynomial $P_n^{(\alpha,\beta)}(x)$ [9]:

$$Y_{jmq}(\theta, \varphi) = C'(1-x)^{-(m+q)/2}(1+x)^{-(m-q)/2}P_{j+m}^{(-m-q),(-m+q)}(x)\,\mathrm{e}^{\mathrm{i}(m+q)\varphi} \tag{9}$$

where $x = \cos\theta$, and

$$C' = 2^m\sqrt{\frac{(2j+1)(j-m)!(j+m)!}{4\pi(j-q)!(j+q)!}}.$$

After integration

$$\int \mathrm{d}^3x\,\Phi^*_{Njm\mu}(\boldsymbol{r})\boldsymbol{r}\sigma_3\Phi_{Njm\mu}(\boldsymbol{r}) \tag{10}$$

we obtain the selection rules that coincide with results of [2]:

$$\Delta j = 0, \pm 1 \qquad \Delta m = 0, \pm 1. \tag{11}$$

Thus, the parity violating transitions with $\Delta j = 0$ are allowed together with the standard transitions with $\Delta j = \pm 1$.

In the second case, however, among operators commuting with Hamiltonian (8), there are the operator σ_3 and the generalized P inversion operator (5), which, however, do not commute with each other. Choosing the general eigenfunctions of the operators (5) and (8) as

$$\Psi_{Njm\mu}(\boldsymbol{r}) = \begin{pmatrix} \Phi_{Njm\mu}(\boldsymbol{r}) \\ \Phi_{Njm-\mu}(\boldsymbol{r}) \end{pmatrix} \tag{12}$$

we calculate the matrix element of the dipole moment operator

$$\int \mathrm{d}^3x\,\Psi^*_{Njm\mu}(\boldsymbol{r})\boldsymbol{r}I\Psi_{Njm\mu}(\boldsymbol{r}) \tag{13}$$

and integrate over the angular part of (13). We have for instance:

$$\begin{aligned}\int_\Omega \mathrm{d}\Omega\,\Psi^*_{j'm'\mu'}(\theta,\varphi)\cos\theta I\Psi_{jm\mu}(\theta,\varphi) &= C\bigg(\int_\Omega \mathrm{d}\Omega\,Y_{j'm'\mu'}(\theta,\varphi)Y_{100}(\theta,\varphi)Y_{jm\mu}(\theta,\varphi) \\ &\quad \pm\int_\Omega \mathrm{d}\Omega\,Y_{j'm'\mu'}(\theta,\varphi)Y_{100}(\theta,\varphi)Y_{jm-\mu}(\theta,\varphi)\bigg) \\ &= C'\begin{pmatrix} j' & 1 & j \\ -m & 0 & m \end{pmatrix}\left[\begin{pmatrix} j' & 1 & j \\ -\mu & 0 & \mu \end{pmatrix} - \begin{pmatrix} j' & 1 & j \\ \mu & 0 & -\mu \end{pmatrix}\right]\end{aligned} \tag{14}$$

where C and C' are some constants. Taking into account the familiar properties of the 3-j symbols

$$\begin{pmatrix} j' & 1 & j \\ -\mu & 0 & \mu \end{pmatrix} = (-1)^{j'+j+1} \begin{pmatrix} j' & 1 & j \\ \mu & 0 & -\mu \end{pmatrix}$$

we see that the integral (13) is non-zero only for $\Delta j = \pm 1$. In an analogous way, after integration of the angular integrals of $\sin\theta\, e^{\pm i\varphi}$ we obtain the selection rules:

$$\Delta j = \pm 1 \qquad \Delta m = 0, \pm 1 \tag{15}$$

that is to say the dipole transitions with parity violation are absent in this case.

Let us note that the use of the wavefunction (12) in (10) does not modify the selection rules (11) but the value of the integral (1) in this case is increased twofold.

Thus the restoring of the standard selection rules and P invariant description of this system are achieved by means of the gauge group extension to $U(1) \otimes U(1)$. At the same time the hypothesis about the Abelian magnetic charge is not connected with an extension of the symmetry group but based on the transition to the non-trivial fibre-bundle over the spacetime base with the structure group $U(1)$ when the connection (potential) and sections (wavefunctions) cannot be described globally.

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